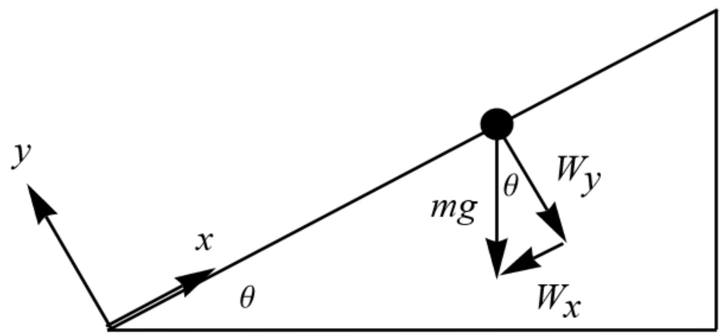
Problem 1.37

A student kicks a frictionless puck with initial speed v_o, so that it slides straight up a plane that is inclined at an angle θ above the horizontal. **(a)** Write down Newton's second law for the puck and solve to give its position as a function of time. **(b)** How long will the puck take to return to its starting point?

Solution

Part (a)

Draw a free-body diagram for the puck. Note that because the puck is frictionless, there's only a gravitational force acting on it.



The components (their magnitudes, rather) of the weight vector along the x-axis and the y-axis are W_x and W_y, respectively.

$$W_x = mg\sin\theta$$

$$W_y = mg\cos\theta$$

Newton's second law states that force is equal to mass times acceleration.

$$\sum \mathbf{F} = m\mathbf{a} \quad \Rightarrow \quad \begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \end{cases}$$

Consider the sum of the forces in the x-direction.

$$\sum F_x = -mg\sin\theta = ma_x$$

There's a negative sign because $\mathbf{W}_x$ points in the negative x-direction. Divide both sides by m.

$$-g\sin\theta = a_x$$

Use the fact that acceleration is the second derivative of position.

$$\frac{d^2x}{dt^2} = -g\sin\theta$$

Integrate both sides with respect to time to get the puck's velocity.

$$\frac{dx}{dt} = -gt\sin\theta + C_1 \tag{1}$$

Use the fact that the initial velocity is v_o to determine C_1, that is, $v(0) = v_\text{o}$.

$$\frac{dx}{dt}(0) = -g(0)\sin\theta + C_1 = v_\text{o} \quad \rightarrow \quad C_1 = v_\text{o}$$

As a result, equation (1) becomes

$$\frac{dx}{dt} = -gt\sin\theta + v_\text{o}.$$

Integrate both sides with respect to time once more to get the puck's position.

$$x(t) = -\frac{gt^2}{2}\sin\theta + v_\text{o}t + C_2 \tag{2}$$

Use the fact that the puck starts at the bottom of the incline, that is, $x(0) = 0$.

$$x(0) = -\frac{g(0)^2}{2}\sin\theta + v_\text{o}(0) + C_2 = 0 \quad \rightarrow \quad C_2 = 0$$

Therefore, equation (2) becomes

$$x(t) = -\frac{gt^2}{2}\sin\theta + v_\text{o}t.$$

Part (b)

To find when the puck will return to its starting point, set $x(t) = 0$ and solve for nonzero t.

$$x(t) = -\frac{gt^2}{2}\sin\theta + v_\text{o}t = 0$$

$$t\left(-\frac{gt}{2}\sin\theta + v_\text{o}\right) = 0$$

$$t = 0 \quad \text{or} \quad -\frac{gt}{2}\sin\theta + v_\text{o} = 0$$

$$t = 0 \quad \text{or} \quad \frac{gt}{2}\sin\theta = v_\text{o}$$

Therefore,

$$t = \frac{2v_\text{o}}{g\sin\theta}.$$